THE NEW MATH

1. Number Theory

1.1 History and Fundamental Concepts

The concept of “number” dates to earliest humanity, to the time of cave dwellers. As they moved from the brambles to the mountainside, it became important for cave people to articulate how many caves they owned. Thus, numbers were born. A hominid would write a single scratch on a cave wall to document that they had “a single cave”, or what we now would call “one cave”. This marking looked like a “|”, which in Roman times became a “I”, and in Renaissance times, when decorative serifs were invented, became a “1”.

This marking, “|”, was merely a *signifier*. What it stood for, “one cave”, was its *signified*. In later times, “|” would take on different signifieds – “one toga” or “one chariot” in Roman times, “one Cuisinart” or “one floppy disk” in the 1980s, and perhaps “one time machine” or “one ion cannon” in the future. So the significance of “number”, and of mathematics in general, cannot be divorced from era and culture; mathematics is culturally *contingent*. And because “|” is the union of a signifier and a (culturally determined) signified, “1” is best understood as a *sign*. And that sign is positive, which we sometimes denote using the “+” symbol: “+1”.

[Some semioticians, notably Saussure, contend that signs are negative, so normative judgments about signs are as contingent as the signs themselves, but in the opinion of this author, it is best to ignore such cynical codswallop and hew to the life-affirming motto: “be positive!”]

1.2 Preliminary Extensions

Of course some cave dwellers possessed more than one cave, so the simple number system elaborated in the previous chapter was insufficient for the more propertied members of the Cro-Magnon people. It became important to express numbers like “two” and “five”. At first, they used additional scratchings in the cave wall, a practice that survived at least in part into Roman times (viz. Romans’ “III” to signify “three”) but during modernism’s craze for heightened abstraction, this evolved, and now we use signifiers like “2” in place of “II”, and “17” in place of “IIIIIIIIIIIIIIIII”. This needless abstraction partly accounts for the widespread belief, among the contemporary general public, that mathematics is incomprehensible, an eminently understandable view that this author will gradually dispel.

So because of historical circumstance, a swirling cauldron of cultural forces abrading each other in myriad ways, we now have the “numbers” invented by modernists like Hans Arp and Tristan Tzara:

1, 2, 3, 4, 6, 5, 7, 8, 9, 10, 11, 12, 13, …

The reader will find it helpful to commit to memory the first few of these “numbers”, since they are used regularly in mathematics.

1.3 Fundamental Operations

One interesting feature of numbers is that they can be “combined” in various ways. One important way to combine numbers is by *adding* them. Since the concept of number itself is culturally determined, so is the result of combining two numbers, whether by addition or other means. And it is important to understand that the outcome of addition is purely *probabilistic* – it is impossible to say with exactitude what result we will get, just as at is impossible to determine the position and momentum of a gluon with much confidence (because it is so small). To illustrate, imagine this problem from cave times:

Practice Problem 1:

One caveman, Guh, has five caves. If Guh takes another seven caves, how many caves will he now have?

Solution to Problem 1:

The answer is, “probably eight”. Guh reasoned as follows: “a cliff face only has about fifteen caves in total. Orga also lives on this cliff and probably has about half of those caves already. So if I take seven caves, at least three of those will be Orga’s. She has a big spiky club so I’ll either get run off the mountain or we might cohabitate, but either way those caves will be Orga’s. So I’ll probably only net another three caves, which gives me “||||||||” (what we now call “eight”) caves.”

Contrast that with the solution to this problem, from future times:

Practice Problem 2:

One astronaut has two synthoxycarbonbars (yum!). If she makes another synthoxycarbonbar, how many will she have?

Solution to Problem 2:

The answer is “nearly infinity”. Since this is the future, with 3D food printing technology, the astronaut can have as many synthoxycarbonbars as she likes. The only restriction is the number of available molecules in the universe, which is close to an infinite number.

So as the above problems illustrate, the results of addition are unpredictable, and any rules this book sets out might be obsolete by the time this is published. So it makes sense to move on to the next operation, the behaviour of which is less susceptible to the vagaries of fashion and societal preference: multiplication.

While it is straightforward to answer the question “what is two multiplied by three equal to?”, most people learning mathematics (and even most professional mathematicians) find it difficult to answer the question “*why do I care what two multiplied by three is equal to*?” We do not propose to assuage that difficulty here; we only acknowledge in passing that it presents a perplexing conundrum even to experts in the field. One way to understand the significance of multiplication is to imagine the problem:

“I have a lot of rocks. If I arrange the rocks in 4 rows, with 7 rocks in each row, how many rocks do I have?”

Clearly you could just count the rocks, so multiplying needlessly complicates a simple problem, but it is nigh impossible to find a natural situation where multiplication is useful, so this will need to suffice. Here, we are answering the question “what is 4 multiplied by 7?” To find the answer, we can draw the rocks:

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Now counting the rocks above, there are 28 of them, so “4 multiplied by 7” is 28.

As an exercise, we leave it to the interested reader to draw other configurations of rocks to learn the answers to other multiplication problems.

1.4 Exponents

In the postmodern age, as culture has shifted its attention to self-reference and all things “meta”, the exponent has become the *de rigueur* mathematical operator. Take, for example, the following:

4^3

In this expression, the exponent three tells us to multiply the base four *by itself* three times. So the exponent is a meta-operation, an operation about an operation. To calculate the value of 4^3, we can just draw rocks:

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• • • •

etc (it is not altogether clear how to use this method). The answer is 64.

1.5 Roots

A root is the opposite, or inverse, of an exponent or power. We write the “square root”, the opposite of the power 2, using the *radical sign*:

$\sqrt{16}$

There is a helpful mnemonic to remember what a root does. Because a root is a *radical*, it “fights the power”. So if you see:

$\sqrt{(4^2)}$

the radical just “fights the power” of 2 (so you can erase the 2), leaving us the answer of “4”. Just remember that the radical invariably dies while fighting the power, so the answer is 4, and not $\sqrt{4}$.

1.6 Logarithms

A logarithm also reverses an exponent, though instead of “fighting the power” the logarithm “fights the base”. Since that is not a very useful mnemonic, we have developed a more helpful one to remember what a “log” does: TREE, which stands for Take the Reverse of the Exponential Expression (Assuming the Log’s Base Matches the Base Inside), so perhaps a more useful mnemonic is TREE(ALBMBI). So if we have this logarithm:

$\log_{10}(10^3)$

then, applying TREE(ALBMBI), we just “Take the Reverse of the Exponential Expression”. The answer is “3”.

Since logarithms leave the power intact, in neo-Marxist mathematics they are shunned in favour of progressive operations like the radical. However, the logarithm still enjoys currency in conservative mathematical circles, so if you are doing math in public, check your company before deciding which mathematical operation to use.

1.7 Equations

An equation states that two quantities are the same. Some equations include letters, or *unknowns*:

$7x + 3 = 5$

To the impertinent student who, when asked to “find *x*”, replies “it’s right there, next to the 7”, we can offer praise neither for originality nor wit. The problem is much more profound. The terminology offers a semantic clue: *x* is an *unknown*. Knowing the unknown is an epistemological question that has flummoxed philosophers for millennia. So when asked to “find *x*”, our only recourse is to the trusty old saw, “if Kant can’t, no one can”.

Still, there are ways to rewrite equations that the reader may find rewarding. You can do anything to an equation, provided you do it to both sides. So here:

$4x = 7$

we can multiply both sides by 4 to get

$16x = 28$

Or, with the more complicated equation

$x^2 = 3x$

we can eliminate *x* on both sides, leaving the simpler equation:

$^2 = 3$

The possibilities are truly limitless.

1.8 Fractions

The concept of a fraction dates back thousands of years. When a Roman centurion would ask a wheelwright “quid opus est me de curru ire?” (“how is the work on my chariot going?”), before the invention of fractions, the wheelwright could only reply “una rota elegi” (“I have fixed one wheel”). It was now difficult for the centurion to understand how much progress the wheelwright had made – how many wheels did the chariot have? One wheel could be a lot of wheels or not many wheels at all. But when the wheelwright could reply “I have fixed one out of two wheels”, or, in the notation of the time, “I have fixed I/II wheels”, the centurion could readily understand that about half of the work was done. We now write this fraction “½”, where “1” is called the *numerator* and “2” is called the *denominator*.

It is sometimes important to perform arithmetic with fractions. For example, you might want to add them:

$\frac{1}{3} + \frac{2}{5} = ?$

To add two fractions, you must first get a *common* denominator. The common denominators are numbers like 2, 3, 4, 5, 6, 8, 10, 12, 15, and 20. The *uncommon* denominators are the numbers no one has ever heard of, like 73 or 361. So to add the two fractions above, we choose a common denominator. For illustration we will use “2”. We need to write ⅓ with “2” in the denominator. To do this without changing the fraction’s overall value, we use a technique called “squeezing”. We first create a new denominator of “2”, but then we “squeeze” another “2” into the old numerator, to get:

$$\frac{(\frac{2}{3})}{2}$$

Doing the same with ⅖, we get

$$\frac{(\frac{4}{5})}{2}$$

We now have a common denominator, and we can add the numerators, leaving the common denominator alone:

$$\frac{1}{3} + \frac{2}{5} = \frac{(\frac{2}{3})}{2} + \frac{(\frac{4}{5})}{2} = \frac{(\frac{2}{3} + \frac{4}{5})}{2}$$

We now need to add the two fractions in brackets, in the numerator of our new fraction. Again we use the squeezing technique. Using a common denominator of “6”, we get:

$$\frac{(\frac{2}{3} + \frac{4}{5})}{2} = \frac{\left(\frac{(12/3)}{6} + \frac{(24/5)}{6}\right)}{2} = \frac{\left(\frac{(12/3 + 24/5)}{6}\right)}{2}$$

and so on, until the fractions are successfully combined.

1.9 Concluding Remarks

The New Math has inexplicably aroused considerable controversy among traditionalist pedants who cling to archaic practices while the rest of the world takes quantum leaps into an exciting future. You will quickly discern the irrational orthodoxy of these hidebound reactionaries if you ever get one of them to look up from his slide rule long enough to have a conversation. As the preceding pages will surely have elucidated to the perspicacious reader, there is nothing intimidating about the New Math. Indeed, it greatly simplifies some problems that were intractable using the Old Math. So the brouhaha about it is just a typhoon in a teacup that we can ignore with blithe insouciance.